# SINGLE - PHASE VARIABLE SPEED SHUNT INDUCTION MOTOR

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Study of a single-phase
variable speed shunt

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STUDY OF A SINGLE-PHASE VARIABLE SPEED SHUNT INDUCTION MOTOR.

# A THESIS

PRESENTED BY

REED CALVIN.

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The single-phase shunt induction motor differs from the ordinary type in that its rotor is equipped with a commutator, upon which bear two pairs of short-circuited brushes for each pair of poles on the motor. The brushes of one set are immediately under the center of the poles, and those of the other set are at right angles thereto. Obviously, the effect is a permanent short-circuiting of the armature windings in two planes, one parallel with and the other perpendicular to the flux from the stator field windings.

To determine the speed variation possible with this type of motor, a Westinghouse compensated series motor was altered to conform with the above conditions. The original connections and the alternations made are shown in the

22148

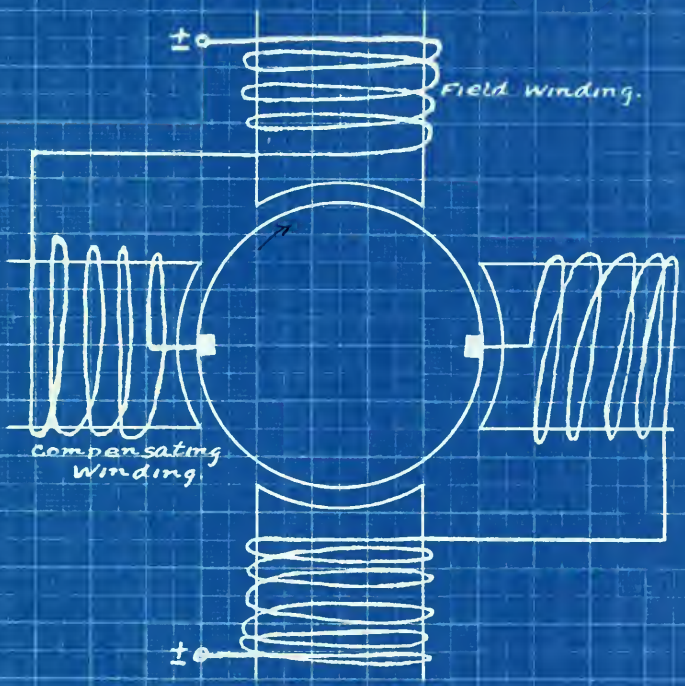
accompanying Fig. 1. The motor is a four-pole machine but a two-pole model is used throughout this discussion to avoid unnecessary complexity, the theory being the same regardless of the No. of poles.

The brushes immediately under the poles, shown in the illustrations as coincident with the Y - axis in rectangular co-ordinates, will be referred to as the Y-brushes, and the other set will be called the X-brushes. The action of the motor is as follows:-

At standstill there is a flux set up parallel to $YY^1$ by the current ($I_f$) through the field windings. Call this flux $\phi_y$.

This flux produces a transformer E.M.F. ($E_y$) with armature, which is a maximum across the brushes $YY^1$ and zero across the brushes $XX^1$.

Straight Series compensated Motor
Original Arrangement of Motor.




Data on Motor
Westinghouse Single-Phase, Series
Machine. Capacity - unrated.
200 Volts. 25 ~. 850 R.P.M.

Fig. 1.

Straight Induction Motor.

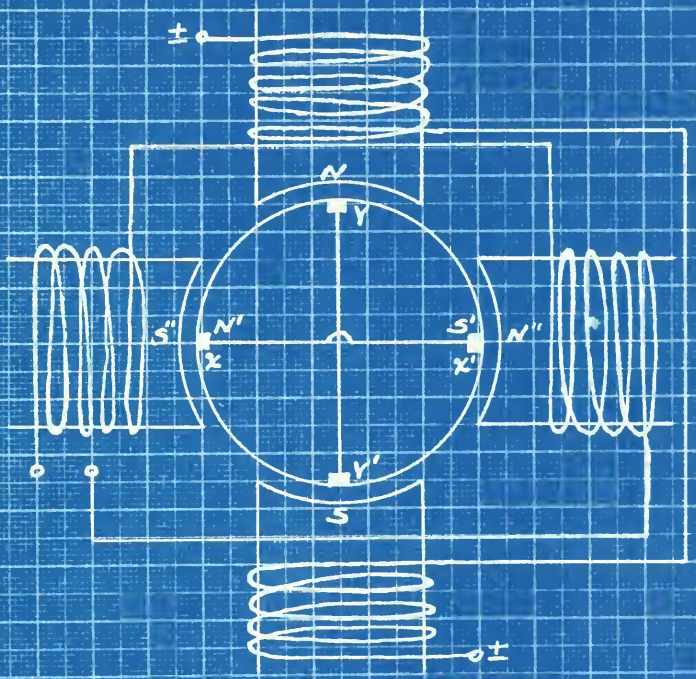



Fig. 1.

## Table 1

| Total Brake lbs | Line Current | $I_m$ K | $I_m$ V K | Line Voltage | Watts Input | R.P.M. | F.T. | Power Factor | Net Brake lbs | Torque Lbs-Ft | H.P. output | H.P. Input | Efficiency % | Slip % |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 13 | 59 | 33 | 38 | 69 | 3550 | 680 | 4070 | .627 | 4.5 | 9 | 1.16 | 3.42 | 34.0 | 9.3 |
| 14 | 62.5 | 34 | 40 | 70 | 3750 | 675 | 4375 | .63 | 5.5 | 11 | 1.41 | 3.7 | 38.1 | 10. |
| 15 | 66. | 33.5 | 45 | 70 | 3050 | 672 | 4625 | .66 | 6.5 | 13 | 1.66 | 4.1 | 40.5 | 10.4 |
| 16 | 69.5 | 34 | 46.5 | 70 | 3200 | 670 | 4860 | .66 | 7.5 | 15 | 1.91 | 4.3 | 44.5 | 10.7 |
| 17.1 | 74. | 33.8 | 51 | 69.3 | 3500 | 662 | 5120 | .685 | 8.6 | 17.2 | 2.16 | 4.7 | 46 | 11.7 |
| 18 | 80.5 | 33.4 | 60 | 69 | 3900 | 655 | 5350 | .705 | 9.5 | 19 | 2.36 | 5.25 | 45 | 12.6 |
| 19 | 83 | 34.2 | 60 | 69 | 4100 | 645 | 5730 | .713 | 10.5 | 21 | 2.58 | 5.5 | 47. | 14.0 |
| 20 | 92 | 34. | 72 | 69.5 | 4700 | 640 | 6400 | .737 | 11.5 | 23 | 2.80 | 6.3 | 44.5 | 14.7 |
| 21 | 100. | 34. | 82 | 69.5 | 5100 | 630 | 6950 | .75 | 12.5 | 25 | 3.00 | 7.0 | 43. | 16. |
| 22 | 105 | 34.8 | 90 | 69. | 5350 | 600 | 7250 | .76 | 13.5 | 27 | 3.08 | 7.2 | 42.8 | 20. |
| 23 | 106 | 36.5 | 88 | 73 | 5650 | 600 | 7750 | .73 | 14.5 | 29 | 3.30 | 7.58 | 43.5 | 20. |
| 0 | 45 | 32. | 10 | 70 | 800 | 715 | 3150 | .254 | 0 | 0 | 0 | 1.07 | 0 | 4.7 |

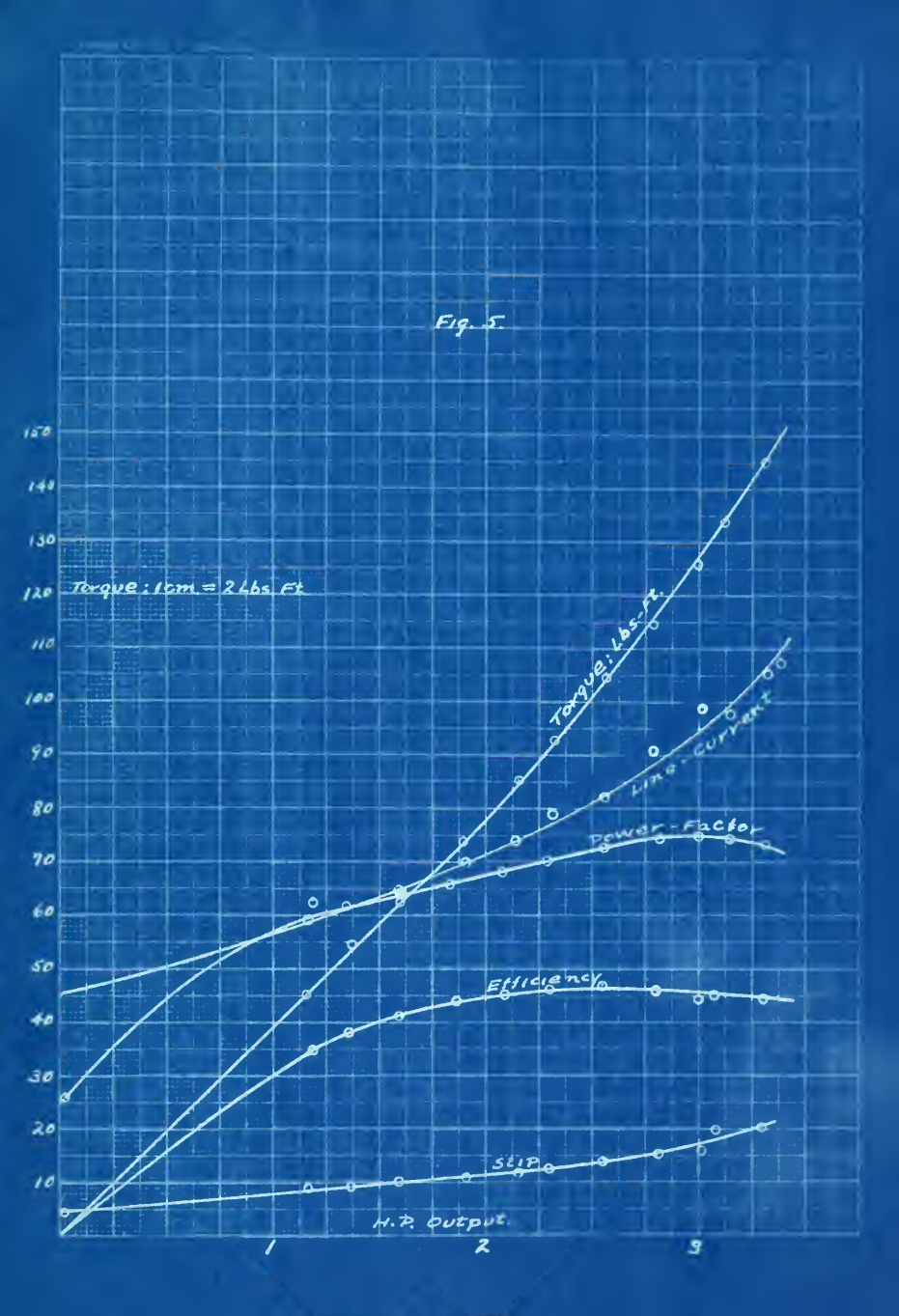


Fig. 5.
Torque: 1cm = 2 lbs. Ft.
Torque: Lbs-ft.
Line-Current
Power-Factor
Efficiency
Slip
H.P. Output.
160
140
130
120
110
100
90
80
70
60
50
40
30
20
10
1
2
3

The machine is then simply a stationary transformer with a short-circuited secondary (the armature). A current ($I_y$) flows between brushes YY1.

If the machine is given a rotation in the direction of the arrow the conductors cut the flux $\phi_y$. An E.M.F. is generated which is in phase with $\phi_y$ and which causes a current $I_x$ to flow in such a direction that its magnetic effect tends to oppose rotation. This speed E.M.F. ($E_x$) is a maximum across the X brushes and zero across the Y brushes. Its current ($I_x$) includes a north pole on the armature, displaced 90 electrical degrees against the direction of rotation from the main field pole whence the flux $\phi_y$ emanates. This This induced north pole on the armature neglecting losses reaches its maximum

value a quarter of a cycle later than the main field pole, since the current $I_x$ lags 90° behind its E.M.F., which E.M.F. is in phase with the main field flux. If a north pole is at the top (as shown in the diagram) a south pole is induced on the armature on the right (equivalent to a north pole in the projecting pole piece shown). This flux at right angles to the main field is called the "speed field", since it is due to a speed generated E.M.F. and is only present when the motor is running.

The rotating conductors also cut the horixontal flux ($\Phi_x$) due to the speed current ($I_x$). As in the case above, a north pole is produced 90 elec. degrees displaced against the direction of rotation and 1/4 of a period behind the pole from which $\Phi_x$ emanates.

This pole is, therefore, located under the main field pole first considered. Since it reaches its maximum strength as a north pole 180 time degrees later than the main field pole, it is at any instant of opposite polarity to the main field pole. Call the flux causing this pole $\Phi_c$ and the corresponding E.M.F. and current $E_c$ and $I_c$, respectively. Since flux flows from the south pole to the north pole in the armature (within a magnet), and from the north pole to the south between the field poles (outside a magnet), it is evident that $\Phi_c$ is in the same direction as the flux from the main field poles. That means that the E.M.F. ($E_c$) causing this flux is in opposition to the E.M.F. ($E_y$) developed in the armature considered as the secondary of the transformer of which the main

field winding is the primary.

$E_c$, the E.M.F. generated by the motion of the conductors through the speed field, is the counter E.M.F. of the motor. Neglecting all losses the free running speed of the motor is such that $E_c$ is equal to $E_y$. Under these conditions the machine is equivalent to a transformer with an open-circuited secondary, and the only current supplied to it is the magnetizing current. This condition is reached at synchronous speed, as shown below:-

(From equation of transformer)

$$E_y = \frac{2\pi f N \phi_y}{\sqrt{2} \times 10^8}$$

where N is the effective turns on the armature, f is the frequency and $\phi_y$ the flux from the field windings which cuts the armature. ($E_y$ is the effective value). If there are C conductors on the armature,

there are $\frac{C}{2}$ turns. These turns form two paths in parallel between the brushes of one set (2 pole model). Hence there are $\frac{C}{4}$ effective turns in series. The E.M.F. generated per coil = Emax Cos $\alpha$ where $E$ max, is the E.M.F. of the coil whose axis is parallel to the flux $\phi_y$ and $\alpha$ is the angle by which the coil considered is displaced from the position of maximum E.M.F. The average E.M.F. per coil = $\frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} E_m \cos\alpha \, d\alpha = \frac{E \max \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}{\pi} = \frac{2}{\pi} E\max.$
That is, the effective turns = $\frac{2}{\pi}$ x the actual turns, or N = $\frac{C}{4} \times \frac{2}{\pi} = \frac{C}{2\pi}$

$$\therefore E_y = \frac{2\pi f \phi_y}{\sqrt{2} \times 10^8} \times \frac{C}{2\pi} = \frac{C f \phi_y}{10^8}$$

The E.M.F. generated by the cutting of the flux $\phi_y$ is

$E_x = \frac{\phi_y C \times n'}{\sqrt{2}\ 10^8}$ ( 2 pole machine). ($E_x$ = effective value)
$n'$ = r.p.m.
At synchronous speed N = f, and hence $E_x$ = $E_y$. $E_x$ is opposed by an E.M.F. of self induction in the windings. This E.M.F.

is(neglecting losses) equal to $E_x$ and hence also to $E_y$. It is generated in the windings by transformer action and therefore must be due to a flux equal to that causing $E_y$, *i.e.*, *has the same value as the mean field* $\Phi_y$, the speed field $(\Phi_k)_\Lambda$ at synchronous speed. Obviously, then, the conductors cutting through the field $\Phi_y$ at synchronous speed also generate an E.M.F., $E_c$, which is equal to Ey, $E_c$, as shown above, is the counter e.m.f. of the motor. The machine will speed up until the counter E. M. F. is equal to the impressed E.M.F. (neglecting losses), and its free running speed is synchronous speed, as with the polyphase induction motor.

The time-phase relations of the various quantities discussed above are shown in the vector diagram, Fig. 2.

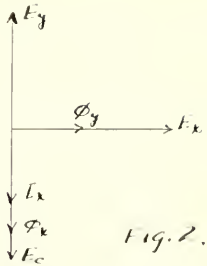



Fig. 2.

This diagram shows $E_c$ directly in opposition to $E_y$. Actually, owing to the resistance of the circuit of which the A-brushes form a part ( the circuit for the speed-field current) these two E. .M.' are not in exact phase opposition, as the diagram below will show.

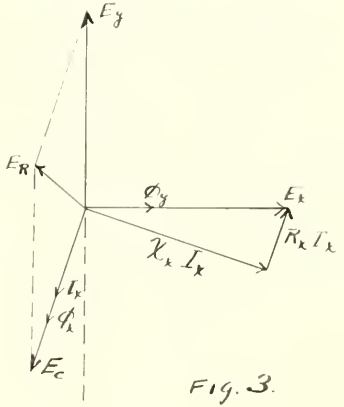



Fig. 3.

$E_x$ is balanced by the vector sum of the $R_x I_x$ and $X_x I_x$ drops, as shown. The flux $\phi_x$ is parallel to the $R_x I_x$ vector, and therefore is 90° displaced from the $X_x I_x$ vector, and more than 90° from the vector $E_x$. Obviously the greater the resistance of the X circuit compared to its reactance, the greater will be the phase angle between $E_y$ and the counter E.M.F., $E_c$. The resultant of $E_y$ and $E_c$ is the E.M.F. which sends current through the Y circuit. The current produced by it reacts upon the speed field flux ( $\phi_x$ ) to produce the torque of the motor. The torque is proportional to the product of the current and the flux, and the cosine of the time phase angle between them. It is apparent, therefore, that a large resistance in the X circuit means a poor torque.

This was the principal trouble found with the motor experimented upon, which was designed for a compensated series motor. It is constructed with resistance leads connecting the coils to the armature in order to keep down the current in the short-circuited coils and prevent sparking at the brushes. The relatively high armature resistance proved detrimental to the torque of the machine as an induction motor. Some improvement of the torque was produced by introducing into the Y brushes an E.M.F. in quadrature with the transformer E.M.F., ($E_y$), tending to throw the resultant, E.M.F. into phase with the flux, $\phi_x$, as shown below.

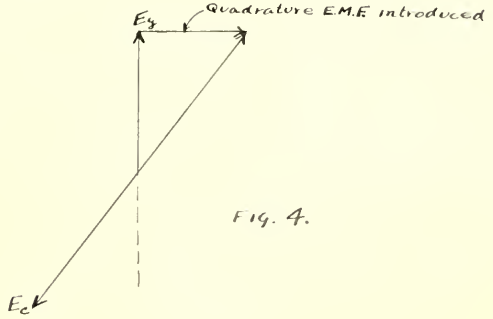



Fig. 4.

The result is to throw $E_c$ and $E_y$ into phase opposition (if the quadrature E.M.F. is of the proper value), so that the resultant E.M.F. tending to send current through the Y circuit is in phase with the flux with which the current reacts (since $\phi_x$ is in phase with $E_c$). At synchronous speed the current is also in phase with $\phi_x$, since the Y circuit has no leakage reactance, as the following facts show:-

A portion of the rotor flux links the rotor conductors only, producing a local field and an E.M.F. of induction in the rotor. Unless this E.M.F. were counterbalanced, a portion of the resultant of $E_y$ and $E_c$ would be necessary to counteract it. At synchronous speed, however, we have exactly the same conditions obtaining in regard to this local field as we had in

the main field. A "speed field" equal to it is set up, and the cutting of this "leakage speed field" produces a counter E.M.F. exactly equal to the E.M.F. due to the leakage flux of the Y circuit (neglecting all losses). Therefore at synchronous speed the leakage reactance is zero.

However, even with the phase compensation, the performance of the motor under load was not very satisfactory. The extremely large current taken made it impossible to load the machine up to the horse power it develops as a series motor. The results of the load tests are shown in Table 1 and Fig.5.

Neglecting the power required to run the motor without load, the free running speed of a motor possessing resistance in the X circuit is reached

when the vector sum of $E_y$ and $E_c$ is at right angles to $E_c$, for at this speed the current is in quadrature with the flux $\phi_k$ and no torque is developed. If the resistance of the X circuit is so small compared to the reactance that its effect in producing phase displacement can be ignored, the resultant of $E_y$ and $E_c$ is equal to $(1-K^2)\ E_t$, where K is the speed with synchronism as unity. The basis for this statement is found in the proof already given of the equality of the transformer E.M.F. and the counter E.M.F. at synchronous speed, losses neglected. That discussion shows that the speed E.M.F. ($E_x$) is equal to the transformer E.M.F. multiplied by the speed expressed in percent of synchronism, or $E_x = k\ E_y$. Also, the counter E.M.F. is equal to the speed E.M.F. multiplied by the speed, or

$E_c = k\ E_x = k^2\ E_y$.

Therefore $E_y - E_c = E_y - k^2\ E_y = (1-k^2)\ E_y$

Similar equations hold with regard to the leakage reactance, and it is therefore apparent that the leakage reactance at any speed k is $(1-k^2)$ times the reactance at standstill. The impedence of the circuit at a speed k is

$$\sqrt{R^2 + (1-k^2)^2 \times X^2}$$

where X is the reactance at standstill. The current at any speed can be considered as made up of the current at synchronous speed (the magnetizing current), plus an additional load current. The E.M.F. effective in forcing the load current through the machine is the resultant of,

$E_y$, $E_c$, R I

(where I is the current at synchronism), and $(1-k^2)$ X I, i.e. it is the resultant

E.M.F. of the motor, ~~means~~ the resistance and reactance drop due to the magnetizing current. The resistance drop caused by the magnetizing current is always perpendicular to $E_c$ and the reactance drop, therefore, is in place with $E_c$. The resultant of $E_y$, $E_c$ and R I is practically equal to $E_y - E_c$, or $(1-k^2)\ E_y$. The reactance drop, $(1-k^2)$ X I, is in phase opposition to $(1-k^2)\ E_y$ and hence is subtracted from it algebraically, giving as the resultant E.M.F. producing the load current $(1-k^2)\ E_y - (1-k^2)$ X I $= E_y -$ XI $(1-k^2)$, $(E_y - XI)$ being constant. It forces current throughca resistance R and a reactance $(1-k^2)$ X. The load current, $I_I$ is, then,

$$I_L = \frac{(E_y - XI)(1-k^2)}{\sqrt{R^2 + (1-k^2)^2 X^2}} = \frac{(E_y - X I)}{\sqrt{\frac{R^2}{(1-k^2)^2} + X^2}}$$

The sine of the phase angle between the current and the E.M.F. is,

$$\sin\theta = \frac{(1-k^2)\,X}{\sqrt{R^2 + (1-k^2)^2\,X^2}} = \frac{X}{\sqrt{\frac{R^2}{(1-k^2)^2} + X^2}}$$

$\therefore I_L = \frac{(E_y - X\,I)}{X} \sin\theta$ , which is the equation of a circle having a diameter

$$\frac{E_y - X\,I}{X}.$$

This forms the basis for the familiar circle diagram of the single-phase induction motor, which will not be entered into here, since the object this thesis is not to investigate the performance of the induction motor, but to study methods of speed variation.

An important point brought out by the current equation is that, although the reaction is actually the variable quantity, the resistance remaining constant, the rotor current acts as if

it flowed in a circuit of constant reactance and variable resistance, and the action is essentially that of a stationary transformer. The load on the motor has exactly the same effect as a resistance load connected to the secondary of a stationary transformer, as is seen from the current equation, where the apparent resistance is seen to be

$\frac{R}{1-k^2}$. It is plain that an increase of load, which lowers the speed (k), produces a decrease of the apparent resistance.

Since we learn by our mistakes, it may not be out of place to call attention here to a fallacious conclusion to which we were led by the vector diagram of the motor.

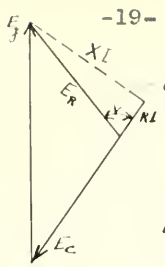



Fig. 6.

Referring to Fig. 6, which represents the main and counter E.M.F.'s of the motor and their resultant, it would seem at first sight that reactance in the Y circuit (through which the current from the resultant ($E_R$) flows) would improve the torque of a motor whose resistance was such as to throw $E_y$ and $E_c$ out of phase, since such reactance would, as the figure shows, tend to throw the current into phase with $E_c$, which is also the phase of the flux $\phi_x$ reacting with the current. A reactance in the Y current, however, failed to produce such an effect, the reason being the following:

Consider that into the circuit of resistance R, at first totally without reactance, a sufficiently large pure reactance is introduced to throw the current vector into place with $E_c$. In the first case the E.M.F. producing current is entirely used up in overcoming resistance, or $E_R = R\ I_1$. The torque is equal to $K\ \phi_k\ I_1 \cos\theta$, where $\phi_k$ is the flux. When the reactance is introduced into the circuit, $E_R \cos\theta = RI_2$. The torque is $K\phi_k I_2$, since the current and the flux are now in phase.

But $\frac{R\ I_2}{R\ I_1} = \frac{E \cos\theta}{E,\ or}$ or $I_2 = I_1 \cos\theta$

Therefore the torque in the second case also is $K\ \phi_k I_2 = K\ \phi_k\ I_1 \cos\theta$. which explains the failure of this attempt to improve the torque of the motor.

From the theory outlined above is is evident that the "field flux" of

the motor, i.e. the flux which reacts to produce the torque, is the speed field flux, $\phi_f$. The X brushes carry the field current of the machine, and the Y brushes carry the load current. It has been shown that the motor runs at such a speed that the counter E.M.F. due to cutting the speed field flux equals the impressed E.M.F. (neglecting losses), ~~first~~ as in a direct current shunt motor. These characteristics give rise to methods of speed control similar to those of a shunt-wound direct-current machine, viz., variation of field current and variation of voltage over the Y-brushes.

Change of Speed by Use of an Inductance in the X Circuit.

If an inductance is inserted in the X circuit, a smaller current will flow for a given speed generated, E.M.F. $E_x$.

Therefore the value of the speed field will become less, and the counter E.M.F. would also decrease in the same proportion as the speed field if the speed of the motor did not change. In order to keep the counter E.M.F. ($E_c$) equal to the transformer E.M.F. ($E_y$) the motor must speed up, increasing the speed field and the counter E.M.F. until balanced E.M.F.'s are obtained.

The diagram (Fig. 7) shows the connections for this method of speed variation. Two values of inductance were used and an increase of speed was obtained of 15.6% and 5.68% respectively. A capacity in the X circuit would increase the speed field current and lower the speed, but the size of condenser needed renders this method of speed control impractical.

Speed Variation by
Inductance in xx Circuit. Method No. 1.

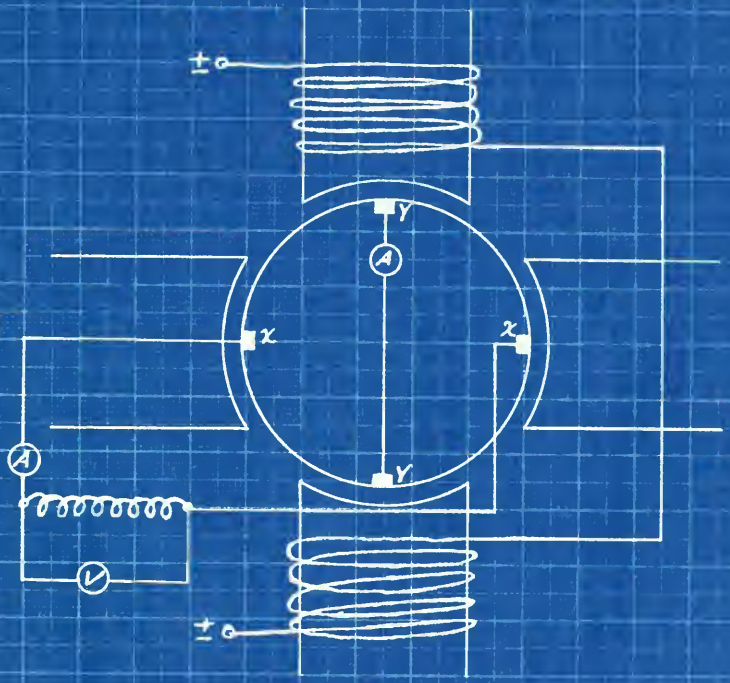



Fig. 7.

Method No. 1. Table 2.

| Current in XX without Inductance | Current in XX with Induct. | Current in YY without Induct. | Current in YY with Induct. | Inductance Drop: Volts |
|---|---|---|---|---|
| 27.2 | 20 | 22 | 18 | 17.5 |
| 30.0 | 27 | 20 | 18 | 8.2 |
| Speed: R.P.M. Without Induct. | Speed with Induct. R.P.M. | | | |
| 705 | 815 | | | |
| 705 | 745 | | | |

Speed Variation by Coils Opposing
or
Strengthing the Speed Field.

If coils, placed so that their flux is parallel to the speed field, are connected in series with the X circuit the speed will be raised or lowered, according as the flux from the coils opposes or aids the speed field flux. Such coils in the motor tested were those which were the main field coils of the machine when run as ~~an induction~~ a series motor. The connections for this method are shown in Fig. 8.

The inductance varies as the square of the number of effective turns, and the field strength varies directly as the ampere turns. If the speed of the motor is to be made one-half of synchronous speed, the speed field must be twice as strong as at synchronous speed in order that the counter E.M.F.

Speed Variation by use of Turns Opposing or Strengthening Speed Field Flux. Method No. 2.

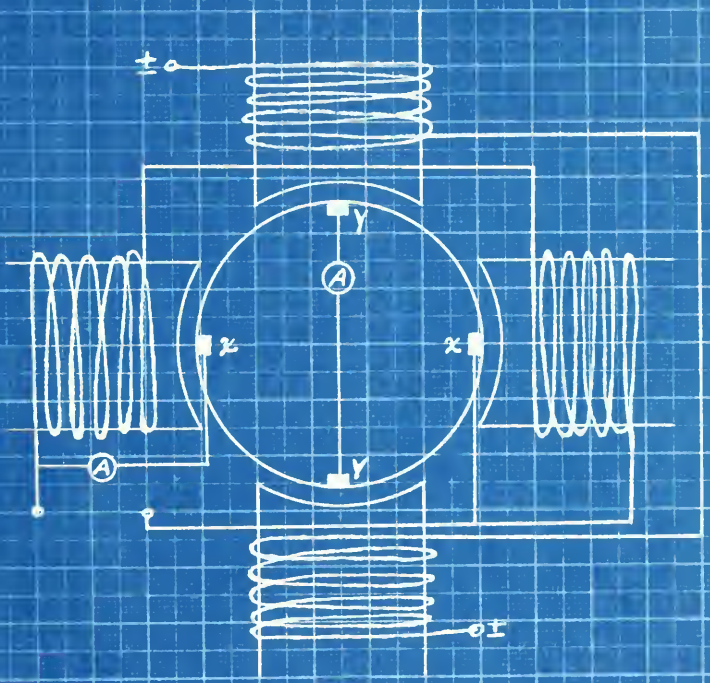



Fig. 8.

Method No. 2. Table 3.

| XX-current without opposing Coils | XX-current with oppo-sing Coils | YY-current without Opposing Coils | YY-current with opposing Coils. |
|---|---|---|---|
| 23 | 51 | 27.5 | 87 |
| Coils aiding 23 | 5 | 27.5 | 17.5 |
| Speed before 700 r.p.m 700 | Speed after 520 r.p.m. 905 | - opposing. - aiding. | |

($E_c$) may equal $E_y$. In general, for a speed k times synchronous the ampere turns of the X circuit (producing the speed field) must be $\frac{1}{k}$ th of their value for synchronous running. The number of turns required for a speed k times synchronism can be thus determined:-

$N_1$ = Effective turns producing speed field at synchronism.

$E_x$ = Effective voltage producing speed field at synchronism.

$N_2$ = Effective turns producing speed field at speed k.

At the new speed the speed field must be $\frac{1}{k}$ th as strong as at synchronism. Assuming constant permeability in the magnetic circuit, the ampere turns must be $\frac{N_1 I_1}{k}$. The voltage producing the speed field will be $kE_x$, since the strength of the Y field is interchanged.

This would give a current of $kI_1$ amperes through the original number of turns. But, as the impedance of the windings is almost wholly reactance, the current through the turns $n_2$ will be $\frac{n_1^2}{n_2^2} \times kI_1$, since the reactance varies at the square of the turns. The ampere turns are $\frac{n1^2}{n_2^2}$ $KI_x$ $n_2$. Equating the two expressions for the ampere turns,

$$\frac{n_1^2}{n_2^2} \; kI_1 = \frac{n_1 I_1}{K} \quad \text{or} \quad \frac{n^1}{n_2} = \frac{1}{k^2}$$

$$n_2 = n_1 k^2.$$

That is, the total turns effective in producing the speed field for a speed k must have $k^2$ times the value of the effective turns on the armature (which are the turns producing the speed field at synchronous speed).

The current for the speed k is

$$\frac{n_1^2}{n_2^2} \times k I_1 = \left(\frac{1}{k}\right)^2 \times k I_1 = \frac{I_1}{k^3}$$

Evidently the use of this method for decreasing the speed is limited by the great increase of the current in the X circuit. For a speed of 1/2 synchronous speed the current must be eight times its normal value.

A speed variation was obtained by this method of from 7[illegible] r.p.m. to 520 r.p.m. with the windings opposing the armature turns, and from 700 to 905 with the windings aiding. As the motor was running with a slip of 60 r.p.m., the ratio of the currents above derived could not be verified experimentally. The large increase in the X-current when the speed was lowered and the decrease when it was raised by this method give an approximate confirmation of the theory.

This number of coils used for varying the speed could not be altered on the motor we tested, but a motor could be constructed in which varying numbers of turns could be used and a rather close speed regulation obtained. It should be noted in this connection that at speeds above synchronism the leakage reactance of the Y-circuit, $(1-k^2)$ X, becomes negative, i.e., is a capacity resistance. This should tend to compensate for the primary leakage reactance of the machine, and improve the power factor. The increased reactance in the X-circuit when the speed-varying coils are strengthening the armature flux tends to improve the torque of the machine at speeds below synchronism by making the effect of the resistance of the X-current negligible compared to the reactance.

Speed Variation by introducing a Voltage over YY-brushes. Method No. 3.

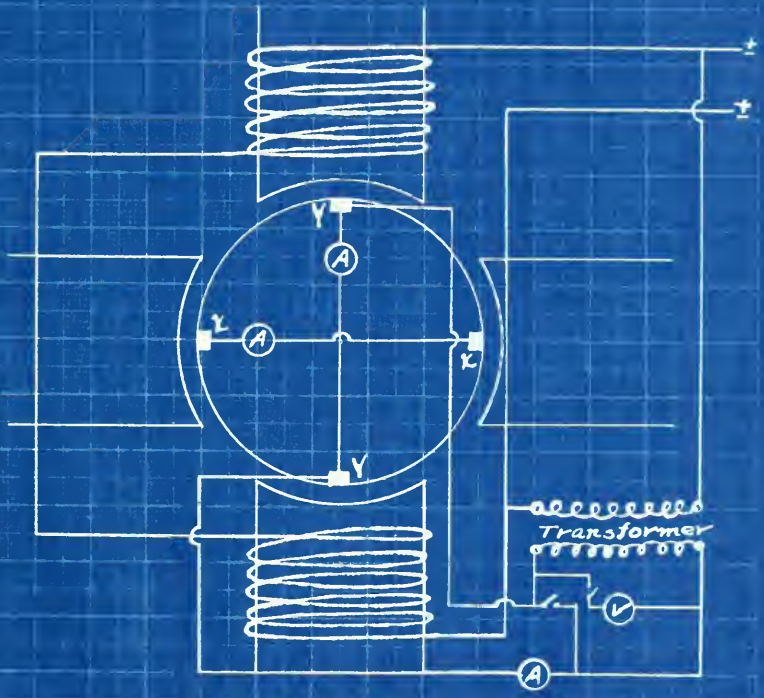



Fig. 9.

Method No. 3. Table 4.

| XX - I before. | XX - I after. | YY - I before | YY - I after | Speed before | Speed after R.P.M. | Impressed E.M.F. | Impressed E.M.F. after | Volts over transformer |
|---|---|---|---|---|---|---|---|---|
| 20.8 | 21 | 23.7 | 24.5 | 690 | 740 | 56 | 55.5 | 5.6 |
| 20.8 | 22 | 23.7 | 25.3 | " | 770 | " | 54 | 10.5 |
| 20.7 | 22.7 | 23.5 | 25.8 | " | 830 | " | 52.5 | 9.4 |
| 20.7 | 23 | 23.5 | 28.5 | " | 885 | " | 50 | 27.2 |
| | | | | | | | | |
| Transformer-connections Reversed. | | | | | | | | |
| 19 | 19 | 20 | 19.8 | 690 | 642 | 56 | 56 | 5.6 |
| 19 | 18.2 | 20 | 18.7 | " | 595 | 56 | 58.5 | 10.2 |
| 20.8 | 17 | 23.2 | 16. | " | 520 | 56.2 | 60. | 18.9 |
| 20.8 | 14 | 23.2 | 15.2 | " | 430 | 56 | 60.5 | 27.5 |

transformer field has no effect upon the speed of the motor. However, an additional voltage applied directly to the Y circuit changes the value of the E.M.F. in that circuit without changing the flux $\phi_f$. Therefore the counter E.M.F. at any speed other than synchronism is equal to k $E_y$, regardless of the additional voltage introduced. If this voltage be called $E_i$, the free-running speed of the motor will be reached when

$$k^2 E_y = E_y + E_i \text{ or}$$

$$k = \sqrt{\frac{E_y + E_i}{E_y}}$$

The change from synchronous speed, then, is seen to be proportional to the square root of the ratio of the voltages after and before the change. Owing to the slip of the motor, only an approximate verification of this fact could be obtained experimentally.

By using an auto-transformer with several taps, speeds of 430, 520, 575, 642, 690 ($E_1 = 0$), 740, 770, 830, and 885 r.p.m. were obtained. As Table 4 shows, these changes were accompanied by no great alteration in the current in either the X-circuit or the Y-circuit. This method appears to be the most satisfactory of the methods used.

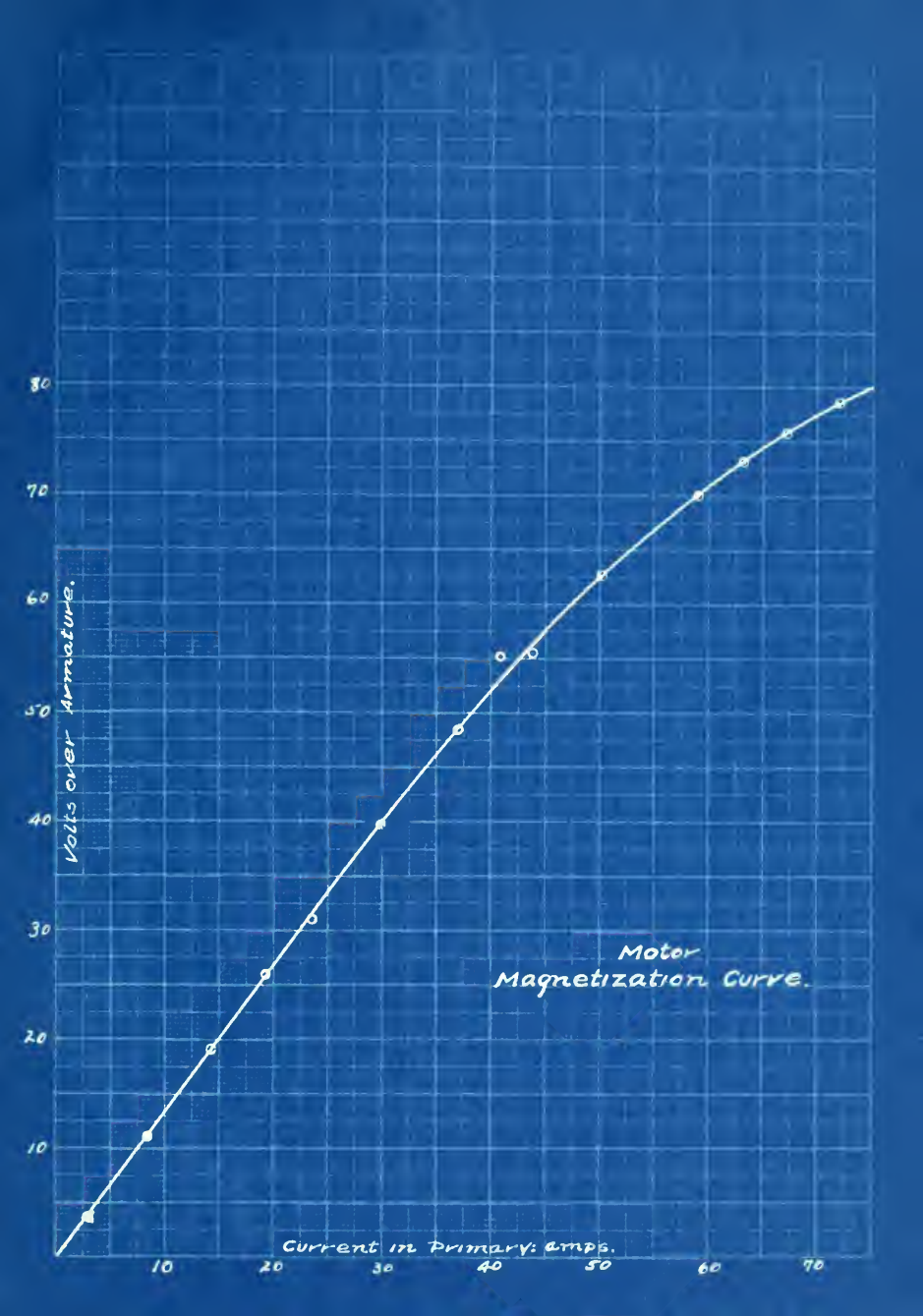
General Conclusions.

Owing to various circumstances, we were unable to start actual experimental work upon the motor until late in the semester; and the short time remaining, together with the poor design of the machine for the purpose, made it impossible to determine how satisfactorily the various methods of speed variation would operate under load conditions. Very little experimenting has been done

on variable speed induction motors of this type, but it would appear, even from our rather meager results, that such machines could be made practicable commercially by proper designing. Since the speed-regulating devices by field control depend on reactance instead of resistance to keep down the current, they should be more efficient than similar regulations on direct current machines. The obvious objection to the variable-speed shunt induction motor is the additional cost, due to the commutator. The simplicity of the squirrel-cage induction motor has been one of the principal reasons for its extensive use. But, even with the additional complexity, there should be a large market for a satisfactory variable-speed alternating current motor in places where direct current machines are now of necessity employed.

# Data for Magnetization curve.

| Volts over Armature | Impressed Volts | Current in Primary | Volts over Armature | Impressed Volts | Current in Primary |
|---|---|---|---|---|---|
| 3.55 | 3.9 | 2.8 | 52.5 | 59. | 40.6 |
| 4.80 | 5.3 | 3.8 | 54. | 63. | 44.1 |
| 7.25 | 8.1 | 5.7 | 59. | 66. | 46.4 |
| 11.05 | 12.3 | 8.7 | 63. | 70.3 | 50. |
| 15.2 | 16.8 | 11.7 | 67. | 75.2 | 54.5 |
| 18.8 | 20.8 | 14.4 | 70. | 78.5 | 58.8 |
| 22.0 | 24.8 | 17.2 | 73. | 81. | 63.1 |
| 25.05 | 28.5 | 19.7 | 76.7 | 86. | 68.6 |
| 27.70 | 30.1 | 21.4 | 78.5 | 89 | 71.8 |
| 31.00 | 35.0 | 24.2 | | | |
| 35.10 | 39.5 | 26.5 | | | |
| 39.20 | 44. | 30. | | | |
| 40.05 | 45. | 30.8 | | | |
| 43.5 | 48.5 | 32.9 | | | |
| 48 | 54.5 | 37. | | | |


Volts over Armature.
80
70
60
50
40
30
20
10
Current in Primary: amps.
10
20
30
40
50
60
70
Motor
Magnetization Curve.